Lithuania Team Selection Test 2006

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by Xixas

1   Let $a_1, a_2, \ldots, a_n$ be positive real numbers, whose sum is 1. Prove that

$$\frac{a_1^2}{a_1 + a_2} + \frac{a_2^2}{a_2 + a_3} + \cdots + \frac{a_{n-1}^2}{a_{n-1} + a_n} + \frac{a_n^2}{a_n + a_1} \geq \frac{1}{2}$$

2   Solve in integers x and y the equation $x^3 - y^3 = 2xy + 8$.

3   Inside a convex quadrilateral $ABCD$ there is a point P such that the triangles PAB, PBC, PCD, PDA have equal areas. Prove that the area of $ABCD$ is bisected by one of the diagonals.

4   Prove that in every polygon there is a diagonal that cuts off a triangle and lies within the polygon.

5   Does the bellow depicted figure fit into a square 5×5.