Iran Team Selection Test 2004

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1 Suppose that p is a prime number. Prove that for each k, there exists an n such that:

$$\left(\frac{n}{p}\right)=\left(\frac{n+k}{p}\right)$$

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2 Suppose that p is a prime number. Prove that the equation $x^2-py^2=-1$ has a solution if and only if $p\equiv 1 \pmod 4$.

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3 Suppose that $ABCD$ is a convex quadrilateral. Let $F=AB\cap CD$, $E=AD\cap BC$ and $T=AC\cap BD$. Suppose that A,B,T,E lie on a circle which intersects with EF at P. Prove that if M is midpoint of AB, then $\angle APM=\angle BPT$.

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4 Let M,M' be two conjugates point in triangle ABC (in the sense that $\angle MAB=\angle M'AC,\dots$). Let P,Q,R,P',Q',R' be foots of perpendiculars from M and M' to BC,CA,AB. Let $E=QR\cap Q'R'$, $F=RP\cap R'P'$ and $G=PQ\cap P'Q'$. Prove that the lines AG,BF,CE are parallel.

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5 This problem is generalization of this one (`http://www.mathlinks.ro/Forum/viewtopic.php?t=5918`).
Suppose G is a graph and $S\subset V(G)$. Suppose we have arbitrarily assign real numbers to each element of S. Prove that we can assign numbers to each vertex in $G\backslash S$ that for each $v\in G\backslash S$ number assigned to v is average of its neighbors.

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6 p is a polynomial with integer coefficients and for every natural n we have $p(n)>n$. x_k is a sequence that: $x_1=1, x_{i+1}=p(x_i)$ for every N one of x_i is divisible by N. Prove that $p(x)=x+1$
