**Iran Team Selection Test 2005**

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**Day 1**

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**1** Suppose that $a_1$, $a_2$, ..., $a_n$ are positive real numbers such that $a_1 \leq a_2 \leq \dots \leq a_n$. Let

$$\frac{a_1+a_2+\dots+a_n}{n}=m; \qquad \frac{a_1^2+a_2^2+\dots+a_n^2}{n}=1.$$

Suppose that, for some $i$, we know $a_i \leq m$. Prove that:

$$n-i \geq n\left(m-a_i\right)^2$$

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**2** Assume $ABC$ is an isosceles triangle that $AB=AC$ Suppose $P$ is a point on extension of side $BC$. $X$ and $Y$ are points on $AB$ and $AC$ that:

$$PX||AC\ ,\ PY||AB$$

Also $T$ is midpoint of arc $BC$. Prove that $PT\perp XY$

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**3** Suppose there are 18 lighthouses on the Persian Gulf. Each of the lighthouses lightens an angle with size 20 degrees. Prove that we can choose the directions of the lighthouses such that whole of the blue Persian (always Persian) Gulf is lightened.

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**Day 2**

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**1** Find all $f: N\longmapsto N$ that there exist $k\in N$ and a prime $p$ that: $\forall n\geq k\ f(n+p)=f(n)$ and also if $m\mid n$ then $f(m+1)\mid f(n)+1$

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**2** Suppose there are $n$ distinct points on plane. There is circle with radius $r$ and center $O$ on the plane. At least one of the points are in the circle. We do the following instructions. At each step we move $O$ to the baricenter of the point in the circle. Prove that location of $O$ is constant after some steps.

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**3** Suppose $S=\{1,2,\dots,n\}$ and $n\geq 3$. There is $f:S^k\longmapsto S$ that if $a,b\in S^k$ and $a$ and $b$ differ in all of elements then $f(a)\neq f(b)$. Prove that $f$ is a function of one of its elements.

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