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1	Find all 3-digit positive integers that are 34 times the sum of their digits.
2	The plan of a house has the shape of a capital L, obtained by suitably placing side-by-side four squares whose sides are 10 metres long. The external walls of the house are 10 metres high. The roof of the house has six faces, starting at the top of the six external walls, and each face forms an angle of 30° with respect to a horizontal plane. Determine the volume of the house (that is, of the solid delimited by the six external walls, the six faces of the roof, and the base of the house).
3	Let A and B are two points on a plane, and let M be the midpoint of AB. Let r be a line and let R and S be the projections of A and B onto r. Assuming that A, M, and R are not collinear, prove that the circumcircle of triangle AMR has the same radius as the circumcircle of BSM.
4	Find all values of n for which all solutions of the equation $x^3 - 3x + n = 0$ are integers.
5	Prove that if $m = 5^n + 3^n + 1$ is a prime, then 12 divides n.
6	We are given a chessboard with 100 rows and 100 columns. Two squares of the board are said to be adjacent if they have a common side. Initially all squares are white. a) Is it possible to colour an odd number of squares in such a way that each coloured square has an odd number of adjacent coloured squares? b) Is it possible to colour some squares in such a way that an odd number of them have exactly 4 adjacent coloured squares and all the remaining coloured squares have exactly 2 adjacent coloured squares? c) Is it possible to colour some squares in such a way that an odd number of them have exactly 2 adjacent coloured squares and all the remaining coloured squares have exactly 4 adjacent coloured squares?