ITAMO 2003

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1	Find all three digit numbers n which are equal to the number formed by three last digit of n^2.
2	A museum has the shape of a $n \times n$ square divided into n^2 rooms of the shape of a unit square $(n > 1)$. Between every two adjacent rooms (i.e. sharing a wall) there is a door. A night guardian wants to organize an inspection journey through the museum according to the following rules. He starts from some room and, whenever he enters a room, he stays there for exactly one minute and then proceeds to another room. He is allowed to enter a room more than once, but at the end of his journey he must have spent exactly k minutes in every room. Find all n and k for which it is possible to organize such a journey.
3	Let a semicircle is given with diameter AB and centre O and let C be a arbitrary point on the segment OB. Point D on the semicircle is such that CD is perpendicular to AB. A circle with centre P is tangent to the arc BD at F and to the segment CD and AB at E and G respectively. Prove that the triangle ADG is isosceles.
4	There are two sorts of people on an island: *knights*, who always talk truth, and *scoundrels*, who always lie. One day, the people establish a council consisting of 2003 members. They sit around a round table, and during the council each member said: ”Both my neighbors are scoundrels”. In a later day, the council meets again, but one member could not come due to illness, so only 2002 members were present. They sit around the round table, and everybody said: ”Both my neighbors belong to the sort different from mine”. Is the absent member a knight or a scoundrel?
5	In each lattice-point of an $m \times n$ grid and in the centre of each of the formed unit squares a pawn is placed. a) Find all such grids with exactly 500 pawns. b) Prove that there are infinitely many positive integers k for which therer is no grid containing exactly k pawns.
6	Every of n guests invited to a dinner has got an invitation denoted by a number from 1 to n. The guests will be sitting around a round table with n seats. The waiter has decided to derve them according to the following rule. At first, he selects one guest and serves him/her at any place. Thereafter, he selects the guests one by one: having chosen a guest, he goes around the table for the number of seats equal to the preceeding guest’s invitation number (starting

from the seat of the preceeding guest), and serves the guest there.
Find all n for which he can select the guests in such an order to serve all the guests.