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- 1 Two people play the following game: there are 40 cards numbered from 1 to 10 with 4 different signs. At the beginning they are given 20 cards each. Each turn one player either puts a card on the table or removes some cards from the table, whose sum is 15. At the end of the game, one player has a 5 and a 3 in his hand, on the table there's a 9, the other player has a card in his hand. What is it's value?

- 2 Solve $p^n + 144 = m^2$ where $m, n \in \mathbb{N}$ and p is a prime number.

- 3 Let A and B be two distinct points on the circle Γ , not diametrically opposite. The point P , distinct from A and B , varies on Γ . Find the locus of the orthocentre of triangle ABP .

- 4 The squares of an infinite chessboard are numbered $1, 2, \dots$ along a spiral, as shown in the picture. A *rightline* is the sequence of the numbers in the squares obtained by starting at one square at going to the right.
 - a) Prove that exists a rightline without multiples of 3.
 - b) Prove that there are infinitely many pairwise disjoint rightlines not containing multiples of 3.

- 5 Consider the inequality
$$(a_1 + a_2 + \dots + a_n)^2 \geq 4(a_1a_2 + a_2a_3 + \dots + a_na_1).$$
 - a) Find all $n \geq 3$ such that the inequality is true for positive reals.
 - b) Find all $n \geq 3$ such that the inequality is true for reals.

- 6 Alberto and Barbara play the following game. Initially, there are some piles of coins on a table. Each player in turn, starting with Albert, performs one of the two following ways:
 - 1) take a coin from an arbitrary pile;
 - 2) select a pile and divide it into two non-empty piles.The winner is the player who removes the last coin on the table. Determine which player has a winning strategy with respect to the initial state.