

Nordic 1996

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- 1 Show that there exists an integer divisible by 1996 such that the sum of its decimal digits is 1996.

 - 2 Determine all real numbers x , such that $x^n + x^{-n}$ is an integer for all integers n .

 - 3 The circle whose diameter is the altitude dropped from the vertex A of the triangle ABC intersects the sides AB and AC at D and E , respectively ($A \neq D$, $A \neq E$). Show that the circumcenter of ABC lies on the altitude drawn from the vertex A of the triangle ADE , or on its extension.

 - 4 The real-valued function f is defined for positive integers, and the positive integer a satisfies $f(a) = f(1995)$, $f(a + 1) = f(1996)$, $f(a + 2) = f(1997)$, $f(n + a) = \frac{f(n)-1}{f(n)+1}$ for all positive integers n .
 - (i) Show that $f(n + 4a) = f(n)$ for all positive integers n .
 - (ii) Determine the smallest possible a .
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