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by parmenides51

1 Let A be a set of seven positive numbers. Determine the maximal number of triples (x, y, z) of elements of A
satisfying $x < y$ and $x + y = z$.

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2 Let $ABCD$ be a convex quadrilateral. We assume that there exists a point P inside the quadrilateral such that
the areas of the triangles ABP, BCP, CDP, and DAP are equal. Show that at least one of the diagonals of the
quadrilateral bisects the other diagonal.

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3 Let A, B, C, and D be four different points in the plane. Three of the line segments AB, AC, AD, BC, BD, and CD have length a. The other three have length b, where $b > a$. Determine all possible values of the quotient $\frac{b}{a}$.
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4 Let f be a function defined in the set $\{0, 1, 2, ...\}$ of non-negative integers, satisfying $f(2x) = 2f(x), f(4x + 1) = 4f(x) + 3$, and $f(4x - 1) = 2f(2x - 1) - 1$.
Show that f is an injection, i.e. if $f(x) = f(y)$, then $x = y$.
