

Hong kong National Olympiad 1999

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by WakeUp

- 1 Find all positive rational numbers $r \neq 1$ such that $r^{\frac{1}{r-1}}$ is rational.

- 2 Let I be the incentre and O the circumcentre of a non-equilateral triangle ABC . Prove that $\angle AIO \leq 90^\circ$ if and only if $2BC \leq AB + AC$.

- 3 Students have taken a test paper in each of $n \geq 3$ subjects. It is known that in any subject exactly three students got the best score, and for any two subjects exactly one student got the best scores in both subjects. Find the smallest n for which the above conditions imply that exactly one student got the best score in each of the n subjects.

- 4 Determine all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x + yf(x)) = f(x) + xf(y) \quad \text{for all } x, y \in \mathbb{R}$$