Hong kong National Olympiad 2000

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1 Let O be the circumcentre of a triangle ABC with $AB > AC > BC$. Let D be a point on the minor arc BC of the circumcircle and let E and F be points on AD such that $AB \perp OE$ and $AC \perp OF$. The lines BE and CF meet at P. Prove that if $PB = PC + PO$, then $\angle BAC = 30^{\circ}$.

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2 Define $a_1 = 1$ and $a_{n+1} = \frac{a_n}{n} + \frac{n}{a_n}$ for $n \in \mathbb{N}$. Find the greatest integer not exceeding a_{2000} and prove your claim.

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3 Find all prime numbers p and q such that $\frac{(7^p - 2^p)(7^q - 2^q)}{pq}$ is an integer.

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4 Find all positive integers $n \geq 3$ such that there exists an n-gon with vertices on lattice points of the coordinate plane and all sides of equal length.
