Hong kong National Olympiad 2001

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1 A triangle ABC is given. A circle Γ, passing through A, is tangent to side BC at point P and intersects sides AB and AC at M and N respectively. Prove that the smaller arcs MP and NP of Γ are equal iff Γ is tangent to the circumcircle of ΔABC at A.

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2 Find, with proof, all positive integers n such that the equation $x^3 + y^3 + z^3 = nx^2y^2z^2$ has a solution in positive integers.

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3 Let $k \geq 4$ be an integer number. $P(x) \in \mathbb{Z}[x]$ such that $0 \leq P(c) \leq k$ for all $c = 0, 1, ..., k+1$. Prove that $P(0) = P(1) = ... = P(k+1)$.

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4 There are 212 points inside or on a given unit circle. Prove that there are at least 2001 pairs of points having distances at most 1.
