

Hong kong National Olympiad 2002

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- 1 Two circles meet at points A and B . A line through B intersects the first circle again at K and the second circle at M . A line parallel to AM is tangent to the first circle at Q . The line AQ intersects the second circle again at R .

(a) Prove that the tangent to the second circle at R is parallel to AK . (b) Prove that these two tangents meet on KM .

- 2 In conference there $n > 2$ mathematicians. Every two mathematicians communicate in one of the n official languages of the conference. For any three different official languages there exists three mathematicians who communicate with each other in these three languages. Find all n such that this is possible.

- 3 Let $a \geq b \geq c \geq 0$ are real numbers such that $a + b + c = 3$. Prove that $ab^2 + bc^2 + ca^2 \leq \frac{27}{8}$ and find cases of equality.

- 4 Let p be a prime number such that $p \equiv 1 \pmod{4}$. Determine $\sum_{k=1}^{\frac{p-1}{2}} \left\{ \frac{k^2}{p} \right\}$, where $\{x\} = x - [x]$.
