Hong kong National Olympiad 2003

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1 Find the greatest real number K such that for all positive real number u, v, w with $u^2 > 4vw$ we have $(u^2 - 4vw)^2 > K(2v^2 - uw)(2w^2 - uv)$

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2 Let $ABCDEF$ regular hexagon of side length 1 and O is its center. In addition to the sides of the hexagon, line segments from O to the every vertex are drawn, making as total of 12 unit segments. Find the number paths of length 2003 along these segments that star at O and terminate at O.

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3 Let K, L, M, N be the midpoints of sides AB, BC, CD, DA of a cyclic quadrilateral $ABCD$. Prove that the orthocentres of triangles ANK, BKL, CLM, DMN are the vertices of a parallelogram.

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4 Find all integer numbers a, b, c such that $\frac{(a+b)(b+c)(c+a)}{2} + (a+b+c)^3 = 1 - abc$.
