Hong kong National Olympiad 2007

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by horizon

1 Let ABC be a triangle and D be a point on BC such that $AB+BD=AC+CD$. The line AD intersects the incircle of triangle ABC at X and Y where X is closer to A than Y i. Suppose BC is tangent to the incircle at E, prove that:
1) EY is perpendicular to AD;
2) $XD=2IM$ where I is the incentre and M is the midpoint of BC.

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2 is there any polynomial of $deg=2007$ with integer coefficients,such that for any integer n,$f(n),f(f(n)),f(f$
is coprime to each other?

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3 There are 2007 boys and 2007 girls in a middle school,every student can attend no more than 100 academic meetings,if we know any pair of students with different sex attend at least one common meeting.prove that there must be a meeting with at least 11 boys and 11 girls attend.

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4 find all positive integer pairs (m,n),satisfies:
(1)$gcd(m,n)=1$,and $m\le\ 2007$
(2)for any $k=1,2,...2007$,we have $[\frac{nk}{m}]=[\sqrt{2}k]$
