Hong kong National Olympiad 2009

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by horizon

1 let a_n be a sequence of integers,a_1 is odd,and for any positive integer n,we have $n(a_{n+1}-a_n+3)=a_{n+1}+a_n+3$,in addition,we have 2010 divides a_{2009}
find the smallest $n\geq 2$,so that 2010 divides a_n

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2 there are n points on the plane,any two vertex are connected by an edge of red,yellow or green,and any triangle with vertex in the graph contains exactly 2 colours.prove that $n<13$

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3 ABC is a right triangle with $\angle C=90$,CD is perpendicular to AB,and D is the foot,ω is the circumcircle of triangle BCD,ω_1 is a circle inside triangle ACD,tangent to AD and AC at M and N respectively,and ω_1 is also tangent to ω.prove that:
(1)$BD*CN+BC*DM=CD*BM$
(2)$BM=BC$

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4 find all pairs of non-negative integer pairs (m,n),satisfies $107^{56}(m^2-1)+2m+3=\binom{113^{114}}{n}$
