Hong kong National Olympiad 2010

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by horizon

1 Let ABC be an arbitrary triangle. A regular n-gon is constructed outward on the three sides of $\triangle ABC$. Find all n such that the triangle formed by the three centres of the n-gons is equilateral.

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2 Let n be a positive integer. Find the number of sequences $x_1, x_2, \ldots x_{2n-1}, x_{2n}$, where $x_i \in \{-1, 1\}$ for each i, satisfying the following condition: for any integer k and m such that $1 \le k \le m \le n$ then the following inequality holds

$$\left| \sum_{i=2k-1}^{2m} x_i \right| \le \ 2$$

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3 Let n be a positive integer. Let a be an integer such that $\gcd(a, n) = 1$. Prove that

$$\frac{a^{\phi(n)} - 1}{n} = \sum_{i \in R} \frac{1}{ai} \left[\frac{ai}{n} \right] \pmod{n}$$

where R is the reduced residue system of n with each element a positive integer at most n.
