JBMO Shortlist 2010

www.artofproblemsolving.com/community/c542883

by Snakes, Ahiles, YasmineMath, phymaths, tnoodee, YESMAths, MinatoF, parmenides51, den_thewhitelion

–	Algebra
1	The real numbers a, b, c, d satisfy simultaneously the equations $$abc-d=1,\quad bcd-a=2,\quad cda-b=3,\quad dab-c=-6.$$ Prove that $a+b+c+d\neq 0$.
2	**Determine all four digit numbers $\bar{a}\bar{b}\bar{c}\bar{d}$ such that** $$a(a+b+c+d)(a^2+b^2+c^2+d^2)(a^6+2b^6+3c^6+4d^6)=\bar{a}\bar{b}\bar{c}\bar{d}$$
3	Find all pairs (x,y) of real numbers such that $\lvert x\rvert+\lvert y\rvert=1340$ and $x^3+y^3+2010xy=670^3$.
4	Let a,b,c be real positive numbers such that $abc(a+b+c)=3$ Prove that $(a+b)(b+c)(c+a)\geq 8$
5	Let $x,y,z>0$ with $x\leq 2,\ y\leq 3$ and $x+y+z=11$ Prove that $xyz\leq 36$
–	Combinatorics
1	**Problem C.1** There are two piles of coins, each containing 2010 pieces. Two players A and B play a game taking turns (A plays first). At each turn, the player on play has to take one or more coins from one pile or exactly one coin from each pile. Whoever takes the last coin is the winner. Which player will win if they both play in the best possible way?
2	A 9×7 rectangle is tiled with tiles of the two types: L-shaped tiles composed by three unit squares (can be rotated repeatedly with 90°) and square tiles composed by four unit squares. Let $n\geq 0$ be the number of the 2×2 tiles which can be used in such a tiling. Find all the values of n.
–	Geometry

1 **Problem G1**
Consider a triangle ABC with $\angle ACB = 90^\circ$. Let F be the foot of the altitude from C. Circle ω touches the line segment FB at point P, the altitude CF at point Q and the circumcircle of ABC at point R. Prove that points A, Q, R are collinear and $AP = AC$.

---

2 Let ABC be acute-angled triangle . A circle $\omega_1(O_1, R_1)$ passes through points B and C and meets the sides AB and AC at points D and E ,respectively .
Let $\omega_2(O_2, R_2)$ be the circumcircle of triangle ADE . Prove that O_1O_2 is equal to the circumradius of triangle ABC .

---

3 Consider a triangle ABC and let M be the midpoint of the side BC. Suppose $\angle MAC = \angle ABC$ and $\angle BAM = 105^\circ$. Find the measure of $\angle ABC$.

---

4 Let AL and BK be angle bisectors in the non-isosceles triangle ABC (L lies on the side BC, K lies on the side AC). The perpendicular bisector of BK intersects the line AL at point M. Point N lies on the line BK such that LN is parallel to MK. Prove that $LN = NA$.

– Number Theory

---

1 Find all integers n, $n \geq 1$, such that $n \cdot 2^{n+1} + 1$ is a perfect square.

---

2 Find n such that $36^n - 6$ is the product of three consecutive natural numbers

---

AoPS Online AoPS Academy AoPS Beast ACADEMY © 2019 AoPS Incorporated
Art of Problem Solving is an ACS WASC Accredited School.