National Olympiad Second Round 2009
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Day 1 December 5th

1 Find all prime numbers p for which p^3-4p+9 is a perfect square.

2 Let Γ be the circumcircle of a triangle ABC, and let D and E be two points different from the vertices on the sides AB and AC, respectively. Let A' be the second point where Γ intersects the bisector of the angle BAC, and let P and Q be the second points where Γ intersects the lines $A'D$ and $A'E$, respectively. Let R and S be the second points of intersection of the lines AA' and the circumcircles of the triangles APD and AQE, respectively.
Show that the lines $DS,\ ER$ and the tangent line to Γ through A are concurrent.

3 *Alice*, who works for the *Graph County Electric Works*, is commissioned to wire the newly erected utility poles in k days. Each day she either chooses a pole and runs wires from it to as many poles as she wishes, or chooses at most 17 pairs of poles and runs wires between each pair. *Bob*, who works for the *Graph County Paint Works*, claims that, no matter how many poles there are and how *Alice* connects them, all the poles can be painted using not more than 2009 colors in such a way that no pair of poles connected by a wire is the same color. Determine the greatest value of k for which *Bob*'s claim is valid.

Day 2 December 6th

1 Let H be the orthocenter of an acute triangle ABC, and let A_1, B_1, C_1 be the feet of the altitudes belonging to the vertices $A,\ B,\ C$, respectively. Let K be a point on the smaller AB_1 arc of the circle with diameter AB satisfying the condition $\angle HKB = \angle C_1KB$. Let M be the point of intersection of the line segment AA_1 and the circle with center C and radius CL where $KB \cap CC_1 = \{L\}$. Let P and Q be the points of intersection of the line CC_1 and the circle with center B and radius BM. Show that $A,\ K,\ P,\ Q$ are concyclic.

2 Show that

$$\frac{(b+c)(a^4-b^2c^2)}{ab+2bc+ca}+\frac{(c+a)(b^4-c^2a^2)}{bc+2ca+ab}+\frac{(a+b)(c^4-a^2b^2)}{ca+2ab+bc}\geq 0$$

for all positive real numbers $a,\ b,\ c$.

3 If $1<k_1<k_2<...<k_n$ and $a_1, a_2, ..., a_n$ are integers such that for every integer N, $k_i \mid N-a_i$ for some $1\leq i\leq n$, find the smallest possible value of n.