National Olympiad Second Round 2011

www.artofproblemsolving.com/community/c5440

by erkamseker, emregirgin35

1 $n \geq 2$ and $E = \{1, 2, ..., n\}$.$A_1, A_2, ..., A_k$ are subsets of E, such that for all $1 \leq i < j \leq k$ Exactly one of $A_i \cap A_j, A_i' \cap A_j, A_i \cap A_j', A_i' \cap A_j'$ is empty set. What is the maximum possible k?

---

2 Let ABC be a triangle $D \in [BC]$ (different than A and B).E is the midpoint of $[CD]$. $F \in [AC]$ such that $\widehat{FEC} = 90$ and $|AF|.|BC| = |AC|.|EC|$. Circumcircle of ADC intersect $[AB]$ at G different than A.Prove that tangent to circumcircle of AGF at F is touch circumcircle of BGE too.

---

3 x, y, z positive real numbers such that $xyz = 1$
Prove that: $\frac{1}{x+y^{20}+z^{11}} + \frac{1}{y+z^{20}+x^{11}} + \frac{1}{z+x^{20}+y^{11}} \leq 1$

---

4 $a_1 = 5$ and $a_{n+1} = a_n^3 - 2a_n^2 + 2$ for all $n \geq 1$. p is a prime such that $p = 3(mod4)$ and $p|a_{2011} + 1$. Show that $p = 3$.

---

5 Let M and N be two regular polygonic area.Define $K(M, N)$ as the midpoints of segments $[AB]$ such that A belong to M and B belong to N. Find all situations of M and N such that $K(M, N)$ is a regualr polygonic area too.

---

6 Let A and B two countries which inlude exactly 2011 cities.There is exactly one flight from a city of A to a city of B and there is no domestic flights (flights are bi-directional).For every city X (doesn't matter from A or from B), there exist at most 19 different airline such that airline have a flight from X to the another city.For an integer k, (it doesn't matter how flights arranged) we can say that there exists at least k cities such that it is possible to trip from one of these k cities to another with same airline.So find the maximum value of k.
