Turkey Team Selection Test 1989

www.artofproblemsolving.com/community/c5444

by matematikolimpiyati

–	April 2nd
1	Let $\mathbb{Z}^+$ denote the set of positive integers. Find all functions $f:\mathbb{Z}^+\times\mathbb{Z}^+\to\mathbb{Z}^+$ such that - $f(m,m)=m$ - $f(m,k)=f(k,m)$ - $f(m,m+k)=f(m,k)$, for each $m,k\in\mathbb{Z}^+$.
2	A positive integer is called a ”double number” if its decimal representation consists of a block of digits, not commencing with 0, followed immediately by an identical block. So, for instance, 360360 is a double number, but 36036 is not. Show that there are infinitely many double numbers which are perfect squares.
3	Let C_1 and C_2 be given circles. Let A_1 on C_1 and A_2 on C_2 be fixed points. If chord A_1P_1 of C_1 is parallel to chord A_2P_2 of C_2, find the locus of the midpoint of P_1P_2.
4	There is a stone on each square of $n\times n$ chessboard. We gather n^2 stones and distribute them to the squares (again each square contains one stone) such that any two adjacent stones are again adjacent. Find all distributions such that at least one stone at the corners remains at its initial square. (Two squares are adjacent if they share a common edge.)
5	There are $n\geq 2$ weights such that each weighs a positive integer less than n and their total weights is less than $2n$. Prove that there is a subset of these weights such that their total weights is equal to n.
6	The circle, which is tangent to the circumcircle of isosceles triangle ABC ($AB=AC$), is tangent AB and AC at P and Q, respectively. Prove that the midpoint I of the segment PQ is the center of the excircle (which is tangent to BC) of the triangle .