Turkey Team Selection Test 1992

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Day 1

1	Is there 14 consecutive positive integers such that each of these numbers is divisible by one of the prime numbers p where $2 \leq p \leq 11$.
2	The line passing through B is perpendicular to the side AC at E. This line meets the circumcircle of $\triangle ABC$ at D. The foot of the perpendicular from D to the side BC is F. If O is the center of the circumcircle of $\triangle ABC$, prove that BO is perpendicular to EF.
3	$x_1, x_2, \cdots, x_{n+1}$ are posive real numbers satisfying the equation $\frac{1}{1+x_1} + \frac{1}{1+x_2} + \cdots + \frac{1}{1+x_{n+1}} = 1$ Prove that $x_1 x_2 \cdots x_{n+1} \geq n^{n+1}$.

Day 2

1	The feet of perpendiculars from the intersection point of the diagonals of cyclic quadrilateral $ABCD$ to the sides AB, BC, CD, DA are P, Q, R, S, respectively. Prove $PQ + RS = QR + SP$.
2	There are n boxes which is numbere from 1 to n. The box with number 1 is open, and the others are closed. There are m identical balls ($m \geq n$). One of the balls is put into the open box, then we open the box with number 2. Now, we put another ball to one of two open boxes, then we open the box with number 3. Go on until the last box will be open. After that the remaining balls will be randomly put into the boxes. In how many ways this arrangement can be done?
3	A circle with radius 4 and 251 distinct points inside the circle are given. Show that it is possible to draw a circle with radius 1 and containing at least 11 of these points.