Turkey Team Selection Test 1994

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Day 1	April 30th
1	f is a function defined on integers and satisfies $f(x)+f(x+3)=x^2$ for every integer x. If $f(19)=94$, then calculate $f(94)$.
2	Let O be the center and $[AB]$ be the diameter of a semicircle. E is a point between O and B. The perpendicular to $[AB]$ at E meets the semicircle at D. A circle which is internally tangent to the arc $\overset{\frown}{BD}$ is also tangent to $[DE]$ and $[EB]$ at K and C, respectively. Prove that $\widehat{EDC}=\widehat{BDC}$.
3	All sides and diagonals of a 25-gon are drawn either red or white. Show that at least 500 triangles, having all three sides are in same color and having all three vertices from the vertices of the 25-gon, can be found.
Day 2	May 1st
1	Let P,Q,R be points on the sides of $\triangle ABC$ such that $P\in[AB], Q\in[BC], R\in[CA]$ and $\frac{\|AP\|}{\|AB\|}=\frac{\|BQ\|}{\|BC\|}=\frac{\|CR\|}{\|CA\|}=k<\frac{1}{2}$ If G is the centroid of $\triangle ABC$, find the ratio $\frac{Area(\triangle PQG)}{Area(\triangle PQR)}$.
2	Show that positive integers n_i, m_i $(i=1,2,3,\cdots)$ can be found such that $\lim_{i\to\infty}\frac{2^{n_i}}{3^{m_i}}=1$
3	Find all integer pairs (a,b) such that $a\cdot b$ divides a^2+b^2+3.