VIII Iberoamerican Interuniversitary Mathematics Competition - Manaus, Brasil

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by Ozc

Problem 1 Find all functions $f:(0,+\infty)\to(0,+\infty)$ that satisfy (i) $f(xf(y)) = yf(x), \forall x,y>0$, (ii) $\lim_{x\to+\infty} f(x) = 0.$

Problem 2 A boa of size k is a graph with $k+1$ vertices $\{0,1,\dots,k-1,k\}$ and edges only between the vertices i and $i+1$ for $0\leq i<k$. The boa is place in a graph G through a injection of graphs. (This is an injective function form the vertices of the boa to the vertices of the graph in such a way that if there is an edge between the vertices x and y in the boa then there must be an edge between $f(x)$ and $f(y)$ in G).

The Boa can move in the graph G using to type of movement each time. If the boa is initially on the vertices $f(0), f(1), \dots, f(k)$ then it moves in one of the following ways:

(i) It choose v a neighbor of $f(k)$ such that $v\notin\{f(0), f(1), \dots, f(k-1)\}$ and the boa now moves to $f(0), f(1), \dots, f(k)$ with $f'(k)=v$ and $f'(i)=f(i+1)$ for $0\leq i<k$, or

(ii) It choose v a neighbor of $f(0)$ such that $v\notin\{f(1), f(2), \dots, f(k)\}$ and the boa now moves to $f(0), f(1), \dots, f(k)$ with $f'(0)=v$ and $f'(i)=f'(i-1)$ for $0<i\leq k$.

Prove that if G is a connected graph with diameter d, then it is possible to put a size $\lceil d/2\rceil$ boa in G such that the boa can reach any vertex of G.
