Turkey Team Selection Test 1996

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Day 1 March 23rd

1 Let $\prod_{n=1}^{1996}(1+nx^{3^n}) = 1 + a_1x^{k_1} + a_2x^{k_2} + ... + a_mx^{k_m}$

where $a_1, a_1, ..., a_m$ are nonzero and $k_1 < k_2 < ... < k_m$. Find a_{1996}.

2 In a parallelogram $ABCD$ with $\angle A < 90$, the circle with diameter AC intersects the lines CB and CD again at E and F , and the tangent to this circle at A meets the line BD at P . Prove that the points P, E, F are collinear.

3 If $0 = x_1 < x_2 < ... < x_{2n+1} = 1$ are real numbers with $x_{i+1} - x_i \leq h$ for $1 \leq i \leq 2n$, show that $\frac{1-h}{2} < \sum_{i=1}^{n} x_{2i}(x_{2i+1} - x_{2i-1}) \leq \frac{1+h}{2}$

Day 2 March 24th

1 The diagonals AC and BD of a convex quadrilateral $ABCD$ with $S_{ABC} = S_{ADC}$ intersect at E. The lines through E parallel to AD, DC, CB, BA
meet AB, BC, CD, DA at K, L, M, N, respectively. Compute the ratio $\frac{S_{KLMN}}{S_{ABC}}$

2 Find the maximum number of pairwise disjoint sets of the form $S_{a,b} = \{n^2 + an + b | n \in \mathbb{Z}\}$, $a, b \in \mathbb{Z}$.

3 Determine all ordered pairs of positive real numbers (a, b) such that every sequence (x_n) satisfying $\lim_{n\to\infty}(ax_{n+1} - bx_n) = 0$ must have $\lim_{n\to\infty} x_n = 0$.