Turkey Team Selection Test 2001
www.artofproblemsolving.com/community/c5456
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Day 1 March 31st

1 Each one of 2001 children chooses a positive integer and writes down his number and names of some of other 2000 children to his notebook. Let A_c be the sum of the numbers chosen by the children who appeared in the notebook of the child c. Let B_c be the sum of the numbers chosen by the children who wrote the name of the child c into their notebooks. The number $N_c = A_c - B_c$ is assigned to the child c. Determine whether all of the numbers assigned to the children could be positive.

2 A circle touches to diameter AB of a unit circle with center O at T where $OT > 1$. These circles intersect at two different points C and D. The circle through O, D, and C meet the line AB at P different from O. Show that

$$|PA| \cdot |PB| = \frac{|PT|^2}{|OT|^2}.$$

3 For all integers x, y, z, let

$$S(x, y, z) = (xy - xz, yz - yx, zx - zy).$$

Prove that for all integers a, b and c with $abc > 1$, and for every integer $n \geq n_0$, there exists integers n_0 and k with $0 < k \leq abc$ such that

$$S^{n+k}(a, b, c) \equiv S^n(a, b, c) \pmod{abc}.$$

($S^1 = S$ and for every integer $m \geq 1$, $S^{m+1} = S \circ S^m$. $(u_1, u_2, u_3) \equiv (v_1, v_2, v_3) \pmod M \iff u_i \equiv v_i \pmod M (i = 1, 2, 3)$.)

Day 2 April 1st

1 Find all ordered pairs of integers (x, y) such that $5^x = 1 + 4y + y^4$.

2 Let H be the intersection of the altitudes of an acute triangle ABC and D be the midpoint of $[AC]$. Show that DH passes through one of the intersection point of the circumcircle of ABC and the circle with diameter $[BH]$.

3 Show that there is no continuous function $f : \mathbb{R} \to \mathbb{R}$ such that for every real number x

$$f(x - f(x)) = \frac{x}{2}.$$

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