Turkey Team Selection Test 2002

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by xeroxia

Day 1 April 6th

1 If $ab(a+b)$ divides a^2+ab+b^2 for different integers a and b, prove that

$$|a-b| > \sqrt[3]{ab}.$$

2 In a triangle ABC, the angle bisector of $\widehat{ABC}$ meets $[AC]$ at D, and the angle bisector of $\widehat{BCA}$ meets $[AB]$ at E. Let X be the intersection of the lines BD and CE where $|BX| = \sqrt{3}|XD|$ ve $|XE| = (\sqrt{3}-1)|XC|$. Find the angles of triangle ABC.

3 A positive integer n and real numbers $a_1, \ldots, a_n$ are given. Show that there exists integers m and k such that

$$|\sum_{i=1}^{m} a_i - \sum_{i=m+1}^{n} a_i| \leq |a_k|.$$

Day 2 April 7th

1 If a function f defined on all real numbers has at least two centers of symmetry, show that this function can be written as sum of a linear function and a periodic function.

[For every real number x, if there is a real number a such that $f(a-x)+f(a+x)=2f(a)$, the point $(a, f(a))$ is called a center of symmetry of the function f.]

2 Two circles are internally tangent at a point A. Let C be a point on the smaller circle other than A. The tangent line to the smaller circle at C meets the bigger circle at D and E; and the line AC meets the bigger circle at A and P. Show that the line PE is tangent to the circle through A, C, and E.

3 Consider $2n+1$ points in space, no four of which are coplanar where $n>1$. Each line segment connecting any two of these points is either colored red, white or blue. A subset M of these points is called a *connected monochromatic* subset, if for each $a, b \in M$, there are points $a = x_0, x_1, \ldots, x_l = b$ that belong to M such that the line segments $x_0x_1, x_1x_2, \ldots, x_{l-1}x_1$ are all have the same color. No matter how the points are colored, if there always exists a connected monochromatic $k-$subset, find the largest value of k. ($l>1$)