Turkey Team Selection Test 2005

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Day 1 April 2nd

1 Find all functions $f : \mathbb{R}_0^+ \mapsto \mathbb{R}_0^+$ satisfying the conditions $4f(x) \geq 3x$ and $f(4f(x) - 3x) = x$ for all $x \geq 0$.

2 Let N be midpoint of the side AB of a triangle ABC with $\angle A$ greater than $\angle B$. Let D be a point on the ray AC such that $CD = BC$ and P be a point on the ray DN which lies on the same side of BC as A and satisfies the condition $\angle PBC = \angle A$. The lines PC and AB intersect at E, and the lines BC and DP intersect at T. Determine the value of $\frac{BC}{TC} - \frac{EA}{EB}$.

3 Initially the numbers 1 through 2005 are marked. A finite set of marked consecutive integers is called a block if it is not contained in any larger set of marked consecutive integers. In each step we select a set of marked integers which does not contain the first or last element of any block, unmark the selected integers, and mark the same number of consecutive integers starting with the integer two greater than the largest marked integer. What is the minimum number of steps necessary to obtain 2005 single integer blocks?

Day 2 April 3rd

1 Show that for any integer $n \geq 2$ and all integers $a_1, a_2, ..., a_n$ the product $\prod_{i<j} (a_j - a_i)$ is divisible by $\prod_{i<j} (j - i)$.

2 Let ABC be a triangle such that $\angle A = 90$ and $\angle B < \angle C$. The tangent at A to its circumcircle Γ meets the line BC at D. Let E be the reflection of A across BC, X the foot of the perpendicular from A to BE, and Y be the midpoint of AX. Let the line BY meet Γ again at Z. Prove that the line BD is tangent to circumcircle of triangle ADZ .

3 We are given 5040 balls in k different colors, where the number of balls of each color is the same. The balls are put into 2520 bags so that each bag contains two balls of different colors. Find the smallest k such that, however the balls are distributed into the bags, we can arrange the bags around a circle so that no two balls of the same color are in two neighboring bags.