Turkey Team Selection Test 2007

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by Litlle 1000t, April

Day 1

1 Find the number of the connected graphs with 6 vertices. (Vertices are considered to be different)

2 Two different points A and B and a circle ω that passes through A and B are given. P is a variable point on ω (different from A and B). M is a point such that MP is the bisector of the angle $\angle APB$ (M lies outside of ω) and $MP = AP + BP$. Find the geometrical locus of M.

3 Let a, b, c be positive reals such that their sum is 1. Prove that

$$\frac{1}{ab+2c^2+2c}+\frac{1}{bc+2a^2+2a}+\frac{1}{ac+2b^2+2b}\geq\frac{1}{ab+bc+ac}.$$

Day 2

1 Let ABC is an acute angled triangle and let $A_1,\ B_1,\ C_1$ are points respectively on $BC,\ CA,\ AB$ such that $\triangle ABC$ is similar to $\triangle A_1B_1C_1$.
Prove that orthocenter of $A_1B_1C_1$ coincides with circumcenter of ABC.

2 A number n is satisfying the conditions below

i) n is a positive odd integer;

ii) there are some odd integers such that their squares' sum is equal to n^4.

Find all such numbers.

3 We write 1 or -1 on each unit square of a 2007×2007 board. Find the number of writings such that for every square on the board the absolute value of the sum of numbers on the square is less then or equal to 1.