Cono Sur Olympiad 1989

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by M4RI0

Day 1

1 Two isosceles triangles with sidelengths x, x, a and x, x, b ($a \neq b$) have equal areas. Find x.

2 Find the sum

$$1 + 11 + 111 + \cdots + \underbrace{111\ldots111}_{n\text{ digits}}.$$

3 A number p is $perfect$ if the sum of its divisors, except p is p. Let f be a function such that: $f(n) = 0$, if n is perfect $f(n) = 0$, if the last digit of n is 4 $f(a.b) = f(a) + f(b)$

Find $f(1998)$

Day 2

1 Let n be square with 4 digits, such that all its digits are less than 6. If we add 1 to each digit the resulting number is another square. Find n

2 Let $ABCD$ be a square with diagonals AC and BD, and P a point in one of the sides of the square. Show that the sum of the distances from P to the diagonals is constant.

3 Show that reducing the dimensions of a cuboid we can't get another cuboid with half the volume and half the surface.