Cono Sur Olympiad 1992

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by Jos

Day 1

1 Find a positive integrer number n such that, if yor put a number 2 on the left and a number 1 on the right, the new number is equal to $33n$.

2 Let P be a point outside the circle C. Find two points Q and R on the circle, such that P,Q and R are collinear and Q is the midpopint of the segmenet PR. (Discuss the number of solutions).

3 Consider the set S of 100 numbers: $1; \frac{1}{2}; \frac{1}{3}; ...; \frac{1}{100}$.
Any two numbers, a and b, are eliminated in S, and the number $a+b+ab$ is added. Now, there are 99 numbers on S.
After doing this operation 99 times, there's only 1 number on S. What values can this number take?

Day 2

1 Prove that there aren't any positive integrer numbers x, y, z such that $x^2+y^2=3z^2$.

2 In a $\triangle ABC$, consider a point E in BC such that $AE \perp BC$. Prove that $AE=\frac{bc}{2r}$, where r is the radio of the circle circumscripte, $b=AC$ and $c=AB$.

3 Consider a $m*n$ board. On each box there's a non-negative integrer number assigned. An operation consists on choosing any two boxes with 1 side in common, and add to this 2 numbers the same integrer number (it can be negative), so that both results are non-negatives.
What conditions must be satisfied initially on the assignment of the boxes, in order to have, after some operations, the number 0 on every box?.