Cono Sur Olympiad 1993

www.artofproblemsolving.com/community/c5473

by José, modeler, parmenides51

Day 1

1 On a table there is a pile with T tokens which incrementally shall be converted into piles with three tokens each. Each step is constituted of selecting one pile removing one of its tokens. And then the remaining pile is separated into two piles. Is there a sequence of steps that can accomplish this process?

a.) $T = 1000$ (Cono Sur)
b.) $T = 2001$ (BWM)

2 Consider a circle with centre O, and 3 points on it, A, B and C, such that $\angle AOB < \angle BOC$. Let D be the midpoint on the arc AC that contains the point B. Consider a point K on BC such that $DK \perp BC$. Prove that $AB + BK = KC$.

3 Find the number of elements that a set B can have, contained in $(1, 2, ..., n)$, according to the following property: For any elements a and b on B ($a \neq b$), $(a - b) \not| (a + b)$.

Day 2

1 On a chess board (8×8) there are written the numbers 1 to 64: on the first line, from left to right, there are the numbers $1, 2, 3, ..., 8$; on the second line, from left to right, there are the numbers $9, 10, 11, ..., 16$;etc. The "$+$" and "$-$" signs are put to each number such that, in each line and in each column, there are 4 "$+$" signs and 4 "$-$" signs. Then, the 64 numbers are added. Find all the possible values of this sum.

2 Prove that there exists a succession $a_1, a_2, ..., a_k, ...$, where each a_i is a digit ($a_i \in (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$) and $a_0 = 6$, such that, for each positive integrer n, the number $x_n = a_0 + 10a_1 + 100a_2 + ... + 10^{n-1}a_{n-1}$ verify that $x_n^2 - x_n$ is divisible by 10^n.

3 Prove that, given a positive integrer n, there exists a positive integrer k_n with the following property: Given any k_n points in the space, 4 by 4 non-coplanar, and associated integrer numbers between 1 and n to each sharp edge that meets 2 of this points, there's necessarily a triangle determined by 3 of them, whose sharp edges have associated the same number.