Cono Sur Olympiad 1994

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Day 1

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1 The positive integrer number n has 1994 digits. 14 of its digits are 0's and the number of times that the other digits: $1, 2, 3, 4, 5, 6, 7, 8, 9$ appear are in proportion $1:2:3:4:5:6:7:8:9$, respectively. Prove that n is not a perfect square.

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2 Consider a circle C with diameter $AB = 1$. A point P_0 is chosen on C, $P_0 \neq A$, and starting in P_0 a sequence of points $P_1, P_2, \dots, P_n, \dots$ is constructed on C, in the following way: Q_n is the symmetrical point of A with respect of P_n and the straight line that joins B and Q_n cuts C at B and P_{n+1} (not necessary different). Prove that it is possible to choose P_0 such that:

i $\angle P_0AB < 1$.

ii In the sequence that starts with P_0 there are 2 points, P_k and P_j, such that $\triangle AP_kP_j$ is equilateral.

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3 Let p be a positive real number given. Find the minimun vale of $x^3 + y^3$, knowing that x and y are positive real numbers such that $xy(x + y) = p$.

Day 2

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1 Pedro and Cecilia play the following game: Pedro chooses a positive integer number a and Cecilia wins if she finds a positive integrer number b, prime with a, such that, in the factorization of $a^3 + b^3$ will appear three different prime numbers. Prove that Cecilia can always win.

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2 Solve the following equation in integers with gcd (x, y) = 1
$x^2 + y^2 = 2z^2$

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3 Consider a $\triangle ABC$, with $AC \perp BC$. Consider a point D on AB such that $CD = k$, and the radius of the inscribe circles on $\triangle ADC$ and $\triangle CDB$ are equals. Prove that the area of $\triangle ABC$ is equal to k^2.
