Cono Sur Olympiad 1995

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by M4RI0

Day 1

1 Find a number with 3 digits, knowing that the sum of its digits is 9, their product is 24 and also the number read from right to left is $\frac{27}{38}$ of the original.

2 There are ten points marked on a circumference, numbered from 1 to 10 and join all points with segments. I color the segments, with red someones and others with blue. Without changing the colors of the segments, renumber all the points from the 1 to the 10. Will be possible to color the segments and to renumber the points so that those numbers that were jointed with red are jointed now with blue and the numbers that were jointed with blue they are jointed now with red?

3 Let $ABCD$ be a rectangle with: $AB = a$, $BC = b$. Inside the rectangle we have to exteriorly tangents circles such that one is tangent to the sides AB and AD,the other is tangent to the sides CB and CD.

1. Find the distance between the centers of the circles(using a and b).
2. When the radiums of both circles change the tangency point between both of them changes, and describes a locus. Find that locus.

Day 2

1 We write the digits of 1995 in the following way:
$199511999955111999999555......$

1. Determine how many digits we have to write such that the sum of the written digits is 2880.
2.Which digit is in position number 1995?

2 The semicircle with centre O and the diameter AC is divided in two arcs AB and BC with ratio $1:3$. M is the midpoint of the radium OC. Let T be the point of arc BC such that the area of the cuadrylateral $OBTM$ is maximum. Find such area in fuction of the radium.

3 Let n be a natural number and $f(n) = 2n - 1995 \lfloor \frac{n}{1000} \rfloor$($\lfloor$ $\rfloor$ denotes the floor function).

1. Show that if for some integer r: $f(f(f...f(n)...)) = 1995$ (where the function f is applied r times), then n is multiple of 1995.

2. Show that if n is multiple of 1995, then there exists r such that: $f(f(f...f(n)...)) = 1995$ (where the function f is applied r times). Determine r if $n = 1995.500 = 997500$

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