Cono Sur Olympiad 2001

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by Shu

Day 1 July 3rd

1 Each entry in a 2000×2000 array is 0, 1, or -1. Show that it's possible for all 4000 row sums and column sums to be distinct.

2 A sequence $a_1, a_2, \ldots$ of positive integers satisfies the following properties.-$a_1 = 1$
-$a_{3n+1} = 2a_n + 1$
-$a_{n+1} \geq a_n$
-$a_{2001} = 200$Find the value of a_{1000}.

Note. In the original statement of the problem, there was an extra condition:-every positive integer appears at least once in the sequence.However, with this extra condition, there is no solution, i.e., no such sequence exists. (Try to prove it.) The problem as written above does have a solution.

3 Three acute triangles are inscribed in the same circle with their vertices being nine distinct points. Show that one can choose a vertex from each triangle so that the three chosen points determine a triangle each of whose angles is at most 90°.

Day 2 July 4th

1 A polygon of area S is contained inside a square of side length a. Show that there are two points of the polygon that are a distance of at least S/a apart.

2 Find all positive integers m for which $2001 \cdot S(m) = m$ where $S(m)$ denotes the sum of the digits of m.

3 A function g defined for all positive integers n satisfies
-$g(1) = 1$;
-for all $n \geq 1$, either $g(n+1) = g(n) + 1$ or $g(n+1) = g(n) - 1$;
-for all $n \geq 1$, $g(3n) = g(n)$; and
-$g(k) = 2001$ for some positive integer k.
Find, with proof, the smallest possible value of k.