Cono Sur Olympiad 2006

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Day 1 May 8th

1 Let $ABCD$ be a convex quadrilateral, let E and F be the midpoints of the sides AD and BC, respectively. The segment CE meets DF in O. Show that if the lines AO and BO divide the side CD in 3 equal parts, then $ABCD$ is a parallelogram.

2 Two players, A and B, play the following game: they retire coins of a pile which contains initially 2006 coins. The players play removing alternatingly, in each move, from 1 to 7 coins, each player keeps the coins that retires. If a player wishes he can pass(he doesn't retire any coin), but to do that he must pay 7 coins from the ones he retired from the pile in past moves. These 7 coins are taken to a separated box and don't interfere in the game any more. The winner is the one who retires the last coin, and A starts the game. Determine which player can win for sure, it doesn't matter how the other one plays. Show the winning strategy and explain why it works.

3 Let n be a natural number. The finite sequence α of positive integer terms, there are n different numbers (α can have repeated terms). Moreover, if from one from its terms any we subtract 1, we obtain a sequence which has, between its terms, at least n different positive numbers. What's the minimum value of the sum of all the terms of α?

Day 2 May 9th

4 Daniel writes over a board, from top to down, a list of positive integer numbers less or equal to 10. Next to each number of Daniel's list, Martin writes the number of times exists this number into the Daniel's list making a list with the same length. If we read the Martin's list from down to top, we get the same
list of numbers that Daniel wrote from top to down. Find the greatest length of the Daniel's list can have.

5 Find all positive integer number n such that $[\sqrt{n}]-2$ divides $n-4$ and $[\sqrt{n}]+2$ divides $n+4$. Note: $[r]$ denotes the integer part of r.

6 We divide the plane in squares shaped of side 1, tracing straight lines parallel bars to the coordinate axles. Each square is painted of black white or. To each as, we recolor all simultaneously squares, in accordance with the following rule: each square Q adopts the color that more appears in the
configuration of five squares indicated in the figure. The recoloration process is repeated in-

definitely.

Determine if exists an initial coloration with black a finite amount of squares such that always has at least one black square, not mattering how many seconds if had passed since the beginning of the process.