Cono Sur Olympiad 2011

www.artofproblemsolving.com/community/c5481

by Leicich

Day 1

1 Find all triplets of positive integers (x, y, z) such that $x^2 + y^2 + z^2 = 2011$.

2 The numbers 1 through 4^n are written on a board. In each step, Pedro erases two numbers a and b from the board, and writes instead the number $\frac{ab}{\sqrt{2a^2+2b^2}}$. Pedro repeats this procedure until only one number remains. Prove that this number is less than $\frac{1}{n}$, no matter what numbers Pedro chose in each step.

3 Let ABC be an equilateral triangle. Let P be a point inside of it such that the square root of the distance of P to one of the sides is equal to the sum of the square roots of the distances of P to the other two sides. Find the geometric place of P.

Day 2

4 A number $\overline{abcd}$ is called *balanced* if $a + b = c + d$. Find all balanced numbers with 4 digits that are the sum of two palindrome numbers.

5 Let ABC be a triangle and D a point in AC. If $\angle CBD - \angle ABD = 60^\circ$, $B\hat{D}C = 30^\circ$ and also $AB \cdot BC = BD^2$, determine the measure of all the angles of triangle ABC.

6 Let Q be a $(2n + 1) \times (2n + 1)$ board. Some of its cells are colored black in such a way that every 2×2 board of Q has at most 2 black cells. Find the maximum amount of black cells that the board may have.