Cono Sur Olympiad 2012

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by Mualpha7

1 1. Around a circumference are written 2012 number, each of with is equal to 1 or -1. If there are not 10 consecutive numbers that sum 0, find all possible values of the sum of the 2012 numbers.

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2 2. In a square $ABCD$, let P be a point in the side CD, different from C and D. In the triangle ABP, the altitudes AQ and BR are drawn, and let S be the intersection point of lines CQ and DR. Show that $\angle ASB = 90$.

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3 3. Show that there do not exist positive integers a, b, c and d, pairwise co-prime, such that $ab+cd$, $ac+bd$ and $ad+bc$ are odd divisors of the number $(a+b-c-d)(a-b+c-d)(a-b-c+d)$.

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4 4. Find the biggest positive integer n, lesser thar 2012, that has the following property:
If p is a prime divisor of n, then p^2-1 is a divisor of n.

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5 5. A and B play alternating turns on a 2012×2013 board with enough pieces of the following types:

Type 1: Piece like Type 2 but with one square at the right of the bottom square.
Type 2: Piece of 2 consecutive squares, one over another.
Type 3: Piece of 1 square.

At his turn, A must put a piece of the type 1 on available squares of the board. B, at his turn, must put exactly one piece of each type on available squares of the board. The player that cannot do more movements loses. If A starts playing, decide who has a winning strategy.

Note: The pieces can be rotated but cannot overlap; they cannot be out of the board. The pieces of the types 1, 2 and 3 can be put on exactly 3, 2 and 1 squares of the board respectively.

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6 6. Consider a triangle ABC with $1 < \frac{AB}{AC} < \frac{3}{2}$. Let M and N, respectively, be variable points of the sides AB and AC, different from A, such that $\frac{MB}{AC} - \frac{NC}{AB} = 1$. Show that circumcircle of triangle AMN pass through a fixed point different from A.
