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1 Determine all triples (x, y, z) of integers greater than 1 with the property that x divides $yz - 1$, y divides $zx - 1$ and z divides $xy - 1$.

2 Twenty-one rectangles of size 3×1 are placed on an 8×8 chessboard, leaving only one free unit square. What position can the free square lie at?

3 A function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies the conditions

$$\begin{cases} f(x + 24) \leq f(x) + 24 \\ f(x + 77) \geq f(x) + 77 \end{cases} \quad \text{for all } x \in \mathbb{R}$$

Prove that $f(x + 1) = f(x) + 1$ for all real x .

4 In a triangle ABC , P and Q are the feet of the altitudes from B and A respectively. Find the locus of the circumcentre of triangle PQC , when point C varies (with A and B fixed) in such a way that $\angle ACB$ is equal to 60° .
