Nordic 1999

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1 The function f is defined for non-negative integers and satisfies the condition $f(n) = f(f(n + 11))$, if $n \leq 1999$ and $f(n) = n - 5$, if $n > 1999$. Find all solutions of the equation $f(n) = 1999$.

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2 Consider 7-gons inscribed in a circle such that all sides of the 7-gon are of different length. Determine the maximal number of 120° angles in this kind of a 7-gon.

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3 The infinite integer plane $Z \times Z = Z^2$ consists of all number pairs (x, y), where x and y are integers. Let a and b be non-negative integers. We call any move from a point (x, y) to any of the points $(x \pm a, y \pm b)$ or $(x \pm b, y \pm a)$ a (a, b)-knight move. Determine all numbers a and b, for which it is possible to reach all points of the integer plane from an arbitrary starting point using only (a, b)-knight moves.

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4 Let $a_1, a_2, ..., a_n$ be positive real numbers and $n \geq 1$. Show that $n(\frac{1}{a_1} + ... + \frac{1}{a_n}) \geq (\frac{1}{1+a_1} + ... + \frac{1}{1+a_n})(n + \frac{1}{a_1} + ... + \frac{1}{a_n})$
When does equality hold?
