JBMO Shortlist 2003

www.artofproblemsolving.com/community/c551170

by Snakes, parmenides51, Iris Aliaj

–	Geometry
1	Is there is a convex quadrilateral which the diagonals divide into four triangles with areas of distinct primes?
2	Is there a triangle with $12\,cm^2$ area and 12 cm perimeter?
3	Let G be the centroid of triangle ABC, and A' the symmetric of A wrt C. Show that G, B, C, A' are concyclic if and only if $GA \perp GC$.
4	Three equal circles have a common point M and intersect in pairs at points A, B, C. Prove that that M is the orthocenter of triangle ABC.
5	Let ABC be an isosceles triangle with $AB = AC$. A semi-circle of diameter $[EF]$ with $E, F \in [BC]$, is tangent to the sides AB, AC in M, N respectively and AE intersects the semicircle at P. Prove that PF passes through the midpoint of $[MN]$.
6	Parallels to the sides of a triangle passing through an interior point divide the inside of a triangle into 6 parts with the marked areas as in the figure. Show that $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} \geq \frac{3}{2}$ `https://cdn.artofproblemsolving.com/attachments/a/a/b0a85df58f2994b0975b654df0c342d8dc4d3` `png`
7	Let D, E, F be the midpoints of the arcs BC, CA, AB on the circumcircle of a triangle ABC not containing the points A, B, C, respectively. Let the line DE meets BC and CA at G and H, and let M be the midpoint of the segment GH. Let the line FD meet BC and AB at K and J, and let N be the midpoint of the segment KJ. a) Find the angles of triangle DMN; b) Prove that if P is the point of intersection of the lines AD and EF, then the circumcenter of triangle DMN lies on the circumcircle of triangle PMN.

© 2022 AoPS Incorporated

Art of Problem Solving is an ACS WASC Accredited School.