Danube Mathematical Olympiad 2017

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–	Juniors
1	What is the smallest value that the sum of the digits of the number $3n^2+n+1, n\in\mathbb{N}$ can take?
2	Let $n\geq 3$ be a positive integer. Consider an $n\times n$ square. In each cell of the square, one of the numbers from the set $M=\{1,2,\dots,2n-1\}$ is to be written. One such filling is called *good* if, for every index $1\leq i\leq n,$ row no. i and column no. $i,$ together, contain all the elements of $M.$ -Prove that there exists $n\geq 3$ for which a good filling exists. -Prove that for $n=2017$ there is no good filling of the $n\times n$ square.
3	Consider an acute triangle ABC in which $A_1, B_1,$ and C_1 are the feet of the altitudes from $A, B,$ and $C,$ respectively, and H is the orthocenter. The perpendiculars from H onto A_1C_1 and A_1B_1 intersect lines AB and AC at P and $Q,$ respectively. Prove that the line perpendicular to B_1C_1 that passes through A also contains the midpoint of the line segment $PQ.$
4	Determine all triples of positive integers (x,y,z) such that $x^4+y^4=2z^2$ and x and y are relatively prime.
–	Seniors
1	Find all polynomials $P(x)$ with integer coefficients such that $a^2+b^2-c^2$ divides $P(a)+P(b)-P(c)$, for all integers a,b,c.
2	Let n be a positive interger. Let n real numbers be wrote on a paper. We call a ”transformation” :choosing 2 numbers a,b and replace both of them with $a*b$. Find all n for which after a finite number of transformations and any n real numbers, we can have the same number written n times on the paper.
3	Let O,H be the circumcenter and the orthocenter of triangle ABC. Let F be the foot of the perpendicular from C onto AB, and M the midpoint of CH. Let N be the foot of the perpendicular from C onto the parallel through H at OM. Let D be on AB such that $CA=CD$. Let BN intersect CD at P. Let PH intersect CA at Q. Prove that $QF \perp OF$.
4	Let us have an infinite grid of unit squares. We write in every unit square a real number, such that the absolute value of the sum of the numbers from any $n*n$ square is less or equal than

1. Prove that the absolute value of the sum of the numbers from any $m * n$ rectangular is less or equal than 4.