

Bangladesh Mathematical Olympiad 2015

www.artofproblemsolving.com/community/c566957

by YanYau, Olympus_mountaineer, Swad

– Secondary Division

1 BdMO National 2015 Secondary Problem 1.

A crime is committed during the hartal. There are four witnesses. The witnesses are logicians and make the following statement:

Witness **One** said exactly one of the four witnesses is a liar.

Witness **Two** said exactly two of the four witnesses is a liar.

Witness **Three** said exactly three of the four witnesses is a liar.

Witness **Four** said exactly four of the four witnesses is a liar.

Assume that each of the statements is either true or false. How many of the witnesses are liars?

2 BdMO National Higher Secondary Problem 3

Let N be the number of pairs of integers (m, n) that satisfies the equation $m^2 + n^2 = m^3$. Is N finite or infinite? If N is finite, what is its value?

3 Let n be a positive integer. Consider the polynomial $p(x) = x^2 + x + 1$. What is the remainder of x^3 when divided by $x^2 + x + 1$. For what positive integers values of n is $x^{2n} + x^n + 1$ divisible by $p(x)$?

Post no: [size=300]100[/size]

4 There are 36 participants at a BdMO event. Some of the participants shook hands with each other. But no two participants shook hands with each other more than once. Each participant recorded the number of handshakes they made. It was found that no two participants with the same number of handshakes made, had shaken hands with each other. Find the maximum possible number of handshakes at the party with proof. (When two participants shake hands with each other, this will be counted as one handshake.)

5 A tetrahedron is a polyhedron composed of four triangular faces. Faces ABC and BCD of a tetrahedron $ABCD$ meet at an angle of $\pi/6$. The area of triangle $\triangle ABC$ is 120. The area of triangle $\triangle BCD$ is 80, and $BC = 10$. What is the volume of the tetrahedron? We call the volume of a tetrahedron as one-third the area of its base times its height.

- 6 Trapezoid $ABCD$ has sides $AB = 92, BC = 50, CD = 19, AD = 70$. AB is parallel to CD . A circle with center P on AB is drawn tangent to BC and AD . Given that $AP = \frac{m}{n}$ (Where m, n are relatively prime). What is $m + n$?
-
- 7 In triangle $\triangle ABC$, the points A', B', C' are on sides BC, AC, AB respectively. Also, AA', BB', CC' intersect at the point O (they are concurrent at O). Also, $\frac{AO}{OA'} + \frac{BO}{OB'} + \frac{CO}{OC'} = 92$. Find the value of $\frac{AO}{OA'} \times \frac{BO}{OB'} \times \frac{CO}{OC'}$.
-