Moldova Team Selection Test 2015

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Day 1

1 Find all functions $f:\mathbb{Z}_+\rightarrow \mathbb{Z}_+$ that satisfy $f(mf(n))=n+f(2015m)$ for all $m,n\in \mathbb{Z}_+$.

2 Consider a triangle $\triangle ABC$, let the incircle centered at I touch the sides BC, CA, AB at points D, E, F respectively. Let the angle bisector of $\angle BIC$ meet BC at M, and the angle bisector of $\angle EDF$ meet EF at N. Prove that A, M, N are collinear.

3 Let p be a fixed odd prime. Find the minimum positive value of $E_p(x,y)=\sqrt{2p}-\sqrt{x}-\sqrt{y}$ where $x,y\in \mathbb{Z}_+$.

4 In how many ways can we color exactly k vertices of an n-gon in red such that any 2 consecutive vertices are not both red. (Vertices are considered to be labeled)

Day 2

1 Let $c\in \Big(0,\dfrac{\pi}{2}\Big), a=\Big(\dfrac{1}{sin(c)}\Big)^{\dfrac{1}{cos^2 (c)}}, b=\Big(\dfrac{1}{cos(c)}\Big)^{\dfrac{1}{sin^2 (c)}}$.
Prove that at least one of a,b is bigger than $\sqrt[11]{2015}$.

2 Let a,b,c be positive real numbers such that $abc=1$. Prove the following inequality:
$a^3+b^3+c^3+\frac{ab}{a^2+b^2}+\frac{bc}{b^2+c^2}+\frac{ca}{c^2+a^2}\geq \frac{9}{2}$.

3 Consider an acute triangle ABC, points E, F are the feet of the perpendiculars from B and C in $\triangle ABC$. Points I and J are the projections of points F, E on the line BC, points K, L are on sides AB, AC respectively such that $IK \parallel AC$ and $JL \parallel AB$. Prove that the lines IE,JF,KL are concurrent.

4 Consider a positive integer n and $A = \{1,2,...,n\}$. Call a subset $X\subseteq A$ ***perfect*** if $|X|\in X$. Call a perfect subset X ***minimal*** if it doesn't contain another perfect subset. Find the number of minimal subsets of A.

Day 3

1 Find all polynomials $P(x)$ with real coefficients which satisfies
$P(2015)=2025$ and $P(x)-10=\sqrt{P(x^2+3)-13}$ for every $x\geq 0$.

2 Prove the equality:
$\tan(\frac{3\pi}{7}) - 4\sin(\frac{\pi}{7}) = \sqrt{7}$.

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3 The tangents to the inscribed circle of $\triangle ABC$, which are parallel to the sides of the triangle and do not coincide with them, intersect the sides of the triangle in the points M, N, P, Q, R, S such that $M, S \in (AB)$, $N, P \in (AC)$, $Q, R \in (BC)$. The interior angle bisectors of $\triangle AMN$, $\triangle BSR$ and $\triangle CPQ$, from points A, B and respectively C have lengths l_1 , l_2 and l_3 .

Prove the inequality: $\frac{1}{l_1^2} + \frac{1}{l_2^2} + \frac{1}{l_3^2} \geq \frac{81}{p^2}$ where p is the semiperimeter of $\triangle ABC$.

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4 Let n and k be positive integers, and let be the sets $X = \{1, 2, 3, ..., n\}$ and $Y = \{1, 2, 3, ..., k\}$. Let P be the set of all the subsets of the set X. Find the number of functions $f : P \to Y$ that satisfy $f(A \cap B) = \min(f(A), f(B))$ for all $A, B \in P$.

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