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by lifeisgood03

–	Day 1
1	A connected graph with at least one vertex of an odd degree is given. Show that one can color the edges of the graph red and blue in such a way that, for each vertex, the absolute difference between the numbers of red and blue edges at that vertex does not exceed 1.
2	Let C be a semicircle with diameter AB. Circles S, S_1, S_2 with radii r, r_1, r_2, respectively, are tangent to C and the segment AB, and moreover S_1 and S_2 are externally tangent to S. Prove that $\frac{1}{\sqrt{r_1}}+\frac{1}{\sqrt{r_2}}=\frac{2\sqrt{2}}{\sqrt{r}}$
3	The cells of an $n\times n$ table ($n\geq 3$) are painted black and white in the chess-like manner. Per move one can choose any 2×2 square and reverse the color of the cells inside it. Find all n for which one can obtain a table with all cells of the same color after finitely many such moves.
4	For a positive integer $A=\overline{a_n...a_1a_0}$ with nonzero digits which are not all the same ($n\geq 0$), the numbers $A_k=\overline{a_{n-k}...a_1a_0a_n...a_{n-k+1}}$ are obtained for $k=1,2,...,n$ by cyclic permutations of its digits. Find all A for which each of the A_k is divisible by A.
–	Day 2
5	Suppose that A and B are sets of real numbers such that $$A\subset B+\alpha\mathbb{Z} \quad \text{and} \quad B\subset A+\alpha\mathbb{Z} \quad \text{for all} \quad \alpha>0$$ (where $X+\alpha$={$x+\alpha n \mid x\in\mathbb{X}, n\in\mathbb{Z}$} (a) Does it follow that $A=B$ (b) The same question, with the assumption that B is bounded
6	At a mathematical olympiad, eight problems were given to 30 contestants. In order to take the difficulty of each problem into account, the jury decided to assign weights to the problems as follows: a problem is worth n points if it was not solved by exactly n contestants. For example, if a problem was solved by all contestants, then it is worth no points. (It is assumed that there are no partial marks for a problem.) Ivan got less points than any other contestant. Find the greatest score he can have.
7	A cube $ABCDA_1B_1C_1D_1$ is given. Find the locus of points E on the face $A_1B_1C_1D_1$ for which there exists a line intersecting the lines AB, A_1D_1, B_1D, and EC.

8 Tom Sawyer must whitewash a circular fence consisting of N planks. He whitewashes the fence going clockwise and following the rule: He whitewashes the first plank, skips two planks, whitewashes one, skips three, and so on. Some planks may be whitewashed several times. Tom believes that all planks will be whitewashed sooner or later, but aunt Polly is sure that some planks will remain unwhitewashed forever. Prove that Tom is right if N is a power of two, otherwise
aunt Polly is right.