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– Day 1

- 1 Pit and Bill play the following game. First Pit writes down a number a , then Bill writes a number b , then Pit writes a number c . Can Pit always play so that the three equations

$$x^3 + ax^2 + bx + c, x^3 + bx^2 + cx + a, x^3 + cx^2 + ax + b$$

have (a) a common real root; (b) a common negative root?

- 2 Find the number of pairs (n, q) , where n is a positive integer and q a non-integer rational number with $0 < q < 2000$, that satisfy $\{q^2\} = \left\{\frac{n!}{2000}\right\}$

- 3 Let $N \geq 5$ be given. Consider all sequences (e_1, e_2, \dots, e_N) with each e_i equal to 1 or -1 . Per move one can choose any five consecutive terms and change their signs. Two sequences are said to be similar if one of them can be transformed into the other in finitely many moves. Find the maximum number of pairwise non-similar sequences of length N .

- 4 The lateral sides and diagonals of a trapezoid intersect a line l , determining three equal segments on it. Must l be parallel to the bases of the trapezoid?

– Day 2

- 5 Nine points are given on a plane, no three of which lie on a line. Any two of these points are joined by a segment. Is it possible to color these segments by several colors in such a way that, for each color, there are exactly three segments of that color and these three segments form a triangle?

- 6 A vertex of a tetrahedron is called perfect if the three edges at this vertex are sides of a certain triangle. How many perfect vertices can a tetrahedron have?

- 7 (a) Find all positive integers n for which the equation $(a^a)^n = b^b$ has a solution in positive integers a, b greater than 1.

(b) Find all positive integers a, b satisfying $(a^a)^5 = b^b$

- 8 To any triangle with side lengths a, b, c and the corresponding angles α, β, γ (measured in radians), the 6-tuple $(a, b, c, \alpha, \beta, \gamma)$ is assigned. Find the minimum possible number n of distinct terms in the 6-tuple assigned to a scalene triangle.