Kosovo Team Selection Test 2015

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1 a)Prove that for every n,natural number exist natural numbers a and b such that $(1-\sqrt{2})^n = a - b\sqrt{2}$ and $a^2 - 2b^2 = (-1)^n$
b)Using first equation prove that for every n exist m such that $(\sqrt{2}-1)^n = \sqrt{m} - \sqrt{m-1}$

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2 Prove that circle l(0,2) with equation $x^2 + y^2 = 4$ contains infinite points with rational coordinates

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3 It's given system of equations $a_{11}x_1 + a_{12}x_2 + a_{1n}x_n = b_1$ $a_{21}x_1 + a_{22}x_2 + a_{2n}x_n = b_2$
..... $a_{n1}x_1 + a_{n2}x_2 + a_{nn}x_n = b_n$
such that $a_{11}, a_{12}, ..., a_{1n}, b_1, a_{21}, a_{22}, ..., a_{2n}, b_2, ..., a_{n1}, a_{n2}, ..., a_{nn}, b_n$, form an arithmetic sequence.If system has one solution find it

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4 Let $P_1, P_2, ..., P_{2556}$ be distinct points inside a regular hexagon $ABCDEF$ of side 1. If any three points from the set $S = \{A, B, C, D, E, F, P_1, P_2..., P_{2556}\}$ aren't collinear, prove that there exists a triangle with area smaller than $\frac{1}{1700}$, with vertices from the set S.

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5 In convex quadrilateral ABCD,diagonals AC and BD intersect at S and are perpendicular.
a)Prove that midpoints M,N,P,Q of AD,AB,BC,CD form a rectangular
b)If diagonals of MNPQ intersect O and AD=5,BC=10,AC=10,BD=11 find value of SO
