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by Morskow

– Day 1

1 Let n be a positive integer. On the table, we have n^2 ornaments in n different colours, not necessarily n of each colour. Prove that we can hang the ornaments on n Christmas trees in such a way that there are exactly n ornaments on each tree and the ornaments on every tree are of at most 2 different colours.

2 Ana and Bojan are playing a game: Ana chooses positive integers a and b and each one gets 2016 pieces of paper, visible to both - Ana gets the pieces with the numbers $a+1$, $a+2$, $\ldots$, $a+2016$ and Bojan gets the pieces with the numbers $b+1$, $b+2$, $\ldots$, $b+2016$ on them. Afterwards, one of them writes the number $a+b$ on the board. In every move, Ana chooses one of her pieces of paper and hands it to Bojan who chooses one of his own, writes their sum on the board and removes them both from the game. When they run out of pieces, they multiply the numbers on the board together. If the result has the same remainder than $a+b$ when divided by 2017, Bojan wins, otherwise, Ana wins. Who has the winning strategy?

3 Let a, b and c be positive real numbers satisfying $abc=1$. Prove that the following inequality holds:

$$\frac{a+b+c}{3} \geq \frac{a}{a^2b+2}+\frac{b}{b^2c+2}+\frac{c}{c^2a+2}.$$

4 Let $\mathcal{K}$ be a circle centered in A. Let p be a line tangent to $\mathcal{K}$ in B and let a line parallel to p intersect $\mathcal{K}$ in C and D. Let the line AD intersect p in E and let F be the intersection of the lines CE and AB. Prove that the line through D, parallel to the tangent through A to the circumcircle of AFD intersects the line CF on $\mathcal{K}$.

5 Let n be a positive integer. We are given a regular $4n$-gon in the plane. We divide its vertices in $2n$ pairs and connect the two vertices in each pair by a line segment. What is the maximal possible amount of distinct intersections of the segments?