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– Paper 1

1 Let x, y be positive integers with $y > 3$ and $x^2 + y^4 = 2((x-6)^2 + (y+1)^2)$. Prove that: $x^2 + y^4 = 1994$.

2 Let A, B, C be collinear points on the plane with B between A and C. Equilateral triangles ABD, BCE, CAF are constructed with D, E on one side of the line AC and F on the other side. Prove that the centroids of the triangles are the vertices of an equilateral triangle, and that the centroid of this triangle lies on the line AC.

3 Find all real polynomials $f(x)$ satisfying $f(x^2) = f(x)f(x-1)$ for all x.

4 Consider all $m \times n$ matrices whose all entries are 0 or 1. Find the number of such matrices for which the number of 1-s in each row and in each column is even.

5 Let $f(n)$ be defined for $n \in \mathbb{N}$ by $f(1) = 2$ and $f(n+1) = f(n)^2 - f(n) + 1$ for $n \geq 1$. Prove that for all $n > 1:$
$1 - \frac{1}{2^{2^{n-1}}} < \frac{1}{f(1)} + \frac{1}{f(2)} + ... + \frac{1}{f(n)} < 1 - \frac{1}{2^{2^n}}$

– Paper 2

1 A sequence (x_n) is given by $x_1 = 2$ and $nx_n = 2(2n-1)x_{n-1}$ for $n > 1$. Prove that x_n is an integer for every $n \in \mathbb{N}$.

2 Let p, q, r be distinct real numbers that satisfy: $q = p(4-p)$, $r = q(4-q)$, $p = r(4-r)$. Find all possible values of $p + q + r$.

3 Prove that for every integer $n > 1$,

$$n((n+1)^{\frac{2}{n}} - 1) < \sum_{i=1}^{n} \frac{2i+1}{i^2} < n(1 - n^{-\frac{2}{n-1}}) + 4.$$

4 Suppose that ω, a, b, c are distinct real numbers for which there exist real numbers x, y, z that satisfy the following equations:
$x + y + z = 1$, $a^2x + b^2y + c^2z = \omega^2$, $a^3x + b^3y + c^3z = \omega^3$, $a^4x + b^4y + c^4z = \omega^4$.

Express ω in terms of a, b, c.

5 If a square is partitioned into n convex polygons, determine the maximum possible number of edges in the obtained figure.

(You may wish to use the following theorem of Euler: If a polygon is partitioned into n polygons with v vertices and e edges in the resulting figure, then $v - e + n = 1$.)