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–	Paper 1
1	Find all pairs of integers (x, y) satisfying $1 + 1996x + 1998y = xy$.
2	For a point M inside an equilateral triangle ABC, let D, E, F be the feet of the perpendiculars from M onto BC, CA, AB, respectively. Find the locus of all such points M for which $\angle FDE$ is a right angle.
3	Find all polynomials $p(x)$ satisfying the equation: $(x-16)p(2x) = 16(x-1)p(x)$ for all x.
4	Let a, b, c be nonnegative real numbers. Suppose that $a+b+c \geq abc$. Prove that: $a^2+b^2+c^2 \geq abc$.
5	Let S be the set of odd integers greater than 1. For each $x \in S$, denote by $\delta(x)$ the unique integer satisfying the inequality $2^{\delta(x)} < x < 2^{\delta(x)+1}$. For $a, b \in S$, define: $a * b = 2^{\delta(a)-1}(b-3)+a$. Prove that if $a, b, c \in S$, then: (a) $a*b \in S$ and (b) $(a*b)*c = a*(b*c)$.
–	Paper 2
1	Given a positive integer n, denote by $\sigma(n)$ the sum of all positive divisors of n. We say that n is $abundant$ if $\sigma(n) > 2n$. (For example, 12 is abundant since $\sigma(12) = 28 > 2 \cdot 12$.) Let a, b be positive integers and suppose that a is abundant. Prove that ab is abundant.
2	A circle Γ is inscribed in a quadrilateral $ABCD$. If $\angle A = \angle B = 120^\circ, \angle D = 90^\circ$ and $BC = 1$, find, with proof, the length of AD.
3	Let A be a subset of $\{0, 1, 2, ..., 1997\}$ containing more than 1000 elements. Prove that either A contains a power of 2 (that is, a number of the form 2^k with $k = 0, 1, 2, ...)$ or there exist two distinct elements $a, b \in A$ such that $a+b$ is a power of 2.
4	Let S be the set of natural numbers n satisfying the following conditions: (i) n has 1000 digits, (ii) all the digits of n are odd, and (iii) any two adjacent digits of n differ by 2. Determine the number of elements of S.

5 Let p be an odd prime number and n a natural number. Then n is called $p-partitionable$ if $T=\{1,2,...,n\}$ can be partitioned into (disjoint) subsets $T_1,T_2,...,T_p$ with equal sums of elements. For example, 6 is 3-partitionable since we can take $T_1=\{1,6\}$, $T_2=\{2,5\}$ and $T_3=\{3,4\}$. (a) Suppose that n is p-partitionable. Prove that p divides n or $n+1$. (b) Suppose that n is divisible by $2p$. Prove that n is p-partitionable.