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–	Paper 1
1	Find all positive integer solutions (a,b,c,n) of the equation: $2^n=a!+b!+c!$.
2	Let ABC be a triangle with sides $BC=a, CA=b, AB=c$ and let D and E be the midpoints of AC and AB, respectively. Prove that the medians BD and CE are perpendicular to each other if and only if $b^2+c^2=5a^2$.
3	Show that if an odd prime number p can be expressed in the form x^5-y^5 for some integers x,y, then: $\sqrt{\frac{4p+1}{5}}=\frac{v^2+1}{2}$ for some odd integer v.
4	Prove that for all positive integers n: $\frac{2n}{3n+1} \leq \sum_{k=n+1}^{2n} \frac{1}{k} \leq \frac{3n+1}{4(n+1)}$.
5	Prove that for all real numbers a,b with $ab>0$ we have: $\sqrt[3]{\frac{a^2b^2(a+b)^2}{4}} \leq \frac{a^2+10ab+b^2}{12}$ and find the cases of equality. Hence, or otherwise, prove that for all real numbers a,b $\sqrt[3]{\frac{a^2b^2(a+b)^2}{4}} \leq \frac{a^2+ab+b^2}{3}$ and find the cases of equality.
–	Paper 2
1	Find the least positive integer a such that 2001 divides $55^n+a\cdot 32^n$ for some odd n.
2	Three hoops are arranged concentrically as in the diagram. Each hoop is threaded with 20 beads, 10 of which are black and 10 are white. On each hoop the positions of the beads are labelled 1 through 20 as shown. We say there is a match at position i if all three beads at position i have the same color. We are free to slide beads around a hoop, not breaking the hoop. Show that it is always possible to move them into a configuration involving no less than 5 matches.
3	In an acute-angled triangle ABC, D is the foot of the altitude from A, and P a point on segment

AD. The lines BP and CP meet AC and AB at E and F respectively. Prove that AD bisects the angle EDF.

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4 Find all nonnegative real numbers x for which $\sqrt[3]{13+\sqrt{x}}+\sqrt[3]{13-\sqrt{x}}$ is an integer.

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5 Determine all functions $f:\mathbb{N}\to\mathbb{N}$ which satisfy:
$f(x+f(y))=f(x)+y$ for all $x,y\in\mathbb{N}$.
